

Impedance match for Stirling type cryocoolers

Wei Dai^{a,*}, Ercang Luo^a, Xiaotao Wang^{a,b}, Zhanghua Wu^a

^a Key Laboratory of Cryogenics, Chinese Academy of Sciences, Zhong Guan Cun Bei Yi Tiao 2, Hai Dian, Beijing 100190, China

^b Graduate University of Chinese Academy of Sciences, Beijing 100049, China

ARTICLE INFO

Article history:

Received 22 October 2010

Received in revised form 30 December 2010

Accepted 5 January 2011

Available online 9 January 2011

Keywords:

Linear compressor

Acoustic impedance

Impedance match

Efficiency

ABSTRACT

Impedance match in Stirling type cryocoolers is important for the compressor efficiency and available acoustic power. This paper generalizes the basic principles concerning the efficiency and acoustic power output of the linear compressor. Starting from basic governing equations and mainly from the viewpoint of energy balance, the physical mechanisms behind the principles are clearly shown. Specially, this paper focuses on the impedance match for an existing compressor, where the current limit and displacement limit should also be taken into consideration when selecting a suitable impedance. Some case studies based on a commercial compressor are also provided for a deep understanding.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

Optimizing the efficiencies of Stirling cryocoolers and Stirling type pulse tube cryocoolers is critical for both space applications and many civil applications. With high frequency oscillating flow inside, these cryocooler systems can be grouped into thermoacoustic systems [1]. From the acoustic viewpoint, the impedance match between the cold-head and compressor is very important for optimizing the efficiency of the compressor as well as maximizing the power capability of the compressor. A few papers have discussed the impedance match for optimizing the efficiency of the compressor [2,3]. However, a systematic discussion about the general rules is lacking, which is the purpose of this paper. The following sections start from the basic governing equations of the linear compressor and perform an in-depth theoretical analysis. Case studies are also provided for a better understanding.

As a matter of fact, there are two different ways when designing the cooler system. One is to design a compressor to match an existing cold-head, which is relatively straightforward but the guiding rule about optimum efficiency should be kept in mind. The other is to find a cold-head to match an existing compressor, which is more important for those who themselves do not have the facility to manufacture a linear compressor and the cost is too high to order a customer-built model. The case studies will mainly focus on the later issue and take the pulse tube cryocooler as an example. A typical illustration of the Stirling type pulse tube cryocooler is

shown in Fig. 1. Note that the same analysis also naturally applies to Stirling cryocoolers.

2. The governing equations and some basics

The governing equations in complex frequency domain for a linear compressor are

$$\hat{E} - BL\hat{u} = \hat{I}(R_e + i\omega L_e) \quad (1)$$

$$\hat{I}BL = \hat{p}A + k\frac{\hat{u}}{i\omega} + R_m\hat{u} + m i\omega\hat{u}, \quad \frac{\hat{u}}{i\omega} = \hat{x} \quad (2)$$

Here $\hat{}$ denotes the complex value containing information on both amplitude and phase of oscillating parameters, which is a conventional method used in the study of alternating electric circuit and acoustics. In Eq. (1), $\hat{E}$ is applied electric voltage, BL is force factor, $\hat{u}$ is the piston velocity, $\hat{I}$ is the current, R_e and L_e are resistance and inductance of the motor coil, ω is angular frequency, i denotes the imaginary part. In Eq. (2), $\hat{p}$ is the pressure wave at the piston front side which drives the cold-head to cool down, A is the piston area, k is spring constant which includes mechanical, magnetic (in case of moving-magnet type motor) and backside gas spring effects, R_m is sometimes called the damping coefficient or the mechanical resistance, m is the moving mass of the compressor and $\hat{x}$ is piston displacement. These equations may seem too simple if we consider many non-linear effects inside the compressor [4], such as the eddy current loss, non-constant BL and clearance losses. Even the equivalent electric circuit may be different in a moving-magnet type motor [5]. However, our purpose here is to build the framework of principles to design a well-matched system and from our

* Corresponding author. Tel.: +86 10 82543751.

E-mail address: dwpeng@yahoo.com (W. Dai).

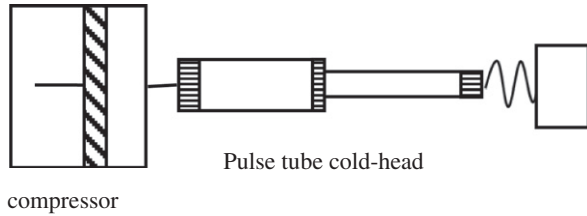


Fig. 1. Illustration of a Stirling type pulse tube cryocooler.

experience, the following analyses based on these equations suffice to provide a useful insight into the mechanism of impedance match and thus lead to a rather good initial design of the practical systems.

Multiplying both sides of Eq. (1) with conjugate of current $\hat{I}^*$, and using $1/2 \text{Re}$ operator (Re means taking real part) to calculate cycle-averaged power flow

$$\begin{aligned} \frac{1}{2} \text{Re}(\hat{E}\hat{I}^*) &= \frac{1}{2} \text{Re}(\hat{I}^* \hat{B} \hat{L} \hat{U}) + \frac{1}{2} \text{Re}(\hat{I}^* (R + i\omega L) \hat{I}) \\ &= \frac{1}{2} \text{Re}(\hat{B} \hat{L} \hat{U} \hat{I}^*) + \frac{1}{2} |\hat{I}|^2 R_e \end{aligned} \quad (3)$$

The physical meaning of this equation is very clear, which is that the electrical power input is partly used for electro-magnetic force to do work and partly dissipated into heat via the resistance. Similarly, multiplying both sides of Eq. (2) with conjugate of piston velocity $\hat{u}^*$ and use $1/2 \text{Re}$ operator, we have

$$\begin{aligned} \frac{1}{2} \text{Re}(\hat{I} \hat{B} \hat{L} \hat{U}^*) &= \frac{1}{2} \text{Re}(\hat{p} \hat{A} \hat{U}^*) + \frac{1}{2} \text{Re} \left(k \frac{\hat{U}}{i\omega} \hat{U}^* \right) + \frac{1}{2} \text{Re}(R_m \hat{U} \hat{U}^*) \\ &\quad + \frac{1}{2} \text{Re}(m i \omega \hat{U} \hat{U}^*) \end{aligned} \quad (4)$$

The second and the fourth term on the right hand side are both equal to zero. Using the thermoacoustic terminology of acoustic impedance

$$Z = \hat{p} / \hat{U} \quad (5)$$

where $\hat{U}$ is volumetric flow rate defined as the piston velocity multiplied by the piston area, Eq. (4) becomes

$$\begin{aligned} \frac{1}{2} \text{Re}(\hat{I} \hat{B} \hat{L} \hat{U}^*) &= \frac{1}{2} \text{Re}(\hat{p} \hat{A} \hat{U}^*) + \frac{1}{2} \text{Re}(R_m \hat{U} \hat{U}^*) \\ &= \frac{1}{2} |\hat{U}|^2 (\text{Re}(ZAA) + R_m) \end{aligned} \quad (6)$$

Here we refer to $\text{Re}(ZAA)$ as the acoustic resistance to distinguish it from the mechanical resistance R_m . The physical meaning of this equation is also very clear: the work done by electro-magnetic force is consumed by the acoustic resistance $\text{Re}(ZAA)$ in the form of acoustic power and the mechanical resistance R_m . There is a delicate difference between the cold-head impedance Z_{ptc} and the impedance Z seen by the compressor piston. The difference could be strongly influenced by the void volume between the cold-head and piston and is important for the impedance match. As seen through this equation, the definition of R_m is also very delicate. Besides the useful work for the cold-head, this equation actually means that many other losses, such as the leakage through the clearance, the friction between piston and cylinder if the clearance is not well kept and the hysteresis losses backside of the compressor, have been generalized into this coefficient.

So from Eqs. (3) and (6), the electrical power input is divided into three parts: one is dissipated in the electric resistance, one is dissipated in the mechanical resistance, and the remaining part goes to the acoustic load for useful work. Properly handling the balance between them is the key to both compressor efficiency and the available acoustic power.

Combining both Eqs. (2) and (3) leads to

$$\begin{aligned} \frac{1}{2} \text{Re}(\hat{E}\hat{I}^*) &= \frac{1}{2} \text{Re} \left(\frac{\hat{I} (BL)^2}{ZAA + k/i\omega + R_m + i\omega m} \hat{I}^* \right) + \frac{1}{2} |\hat{I}|^2 R_e \\ &= \frac{1}{2} |\hat{I}|^2 \frac{(BL)^2 (\text{Re}(ZAA) + R_m)}{(\text{Re}(ZAA) + R_m)^2 + (\text{Im}(ZAA) - k/\omega + \omega m)^2} + \frac{1}{2} |\hat{I}|^2 R_e \end{aligned} \quad (7)$$

I_m means taking the imaginary part. For the cold-head, the imaginary part of the load impedance is negative. As already deduced in Ref. [2], one of the basic requirements for high efficiency of the system is that

$$\text{Im}(ZAA) - k/\omega + \omega m = 0 \quad (8)$$

which means that the mass is mechanically resonated by spring forces with a spring constant equal to $k - \omega \text{Im}(ZAA)$. $-\omega \text{Im}(ZAA)$ is the gas spring at the front side of the piston, which contributes a very large proportion of the whole spring force in a normal design.

Rewriting Eq. (2) into the form

$$\hat{I} BL = (\text{Re}(ZAA) + R_m) \hat{U} + i(\text{Im}(ZAA) - k/\omega + \omega m) \hat{U} \quad (9)$$

And combining it with Eq. (8), we can see that the motor current is in-phase with the velocity of the piston. This in-phase relationship is well known and reflects that in the resonant state the current amplitude is minimized in producing the same amount of work and Joule loss in the motor coil is thus reduced. Off-resonance only means that part of the electro-magnetic force acts as a kind of spring force (either positive or negative) without doing any work. At this mechanically-resonant point, Eq. (7) further reduces to

$$\frac{1}{2} \text{Re}(\hat{E}\hat{I}^*) = \frac{1}{2} |\hat{I}|^2 \frac{(BL)^2}{\text{Re}(ZAA) + R_m} + \frac{1}{2} |\hat{I}|^2 R_e \quad (10)$$

The first item on the right hand side is actually another form of the electro-magnetic work. Starting with these equations, we will discuss how to maximize the compressor efficiency and acoustic power output. Unless otherwise stated, the following discussion is based on the assumption that mechanical resonance is achieved by satisfying Eq. (8).

3. Maximizing the efficiency and acoustic power output of a compressor

3.1. Impedance match for maximizing the efficiency

The compressor efficiency is defined as output acoustic power W_a , i.e., the power dissipated by the acoustic resistance, divided by input electric power W_e .

$$\eta = \frac{W_a}{W_e} = \frac{\frac{1}{2} |\hat{U}|^2 \text{Re}(ZAA)}{\frac{1}{2} \text{Re}(\hat{E}\hat{I}^*)} \quad (11)$$

From Eqs. (6) and (10), it is very clear that the acoustic resistance $\text{Re}(ZAA)$ must be kept within a certain range for a high compressor efficiency. If $\text{Re}(ZAA)$ is too small, although most of the electric power will be converted into electro-magnetic work (i.e. the first item on the right hand side of Eq. (10)), most of the work will be consumed by the mechanical resistance, not by the acoustic load, as expressed through Eq. (6). However, if $\text{Re}(ZAA)$ is too large, then from Eq. (10), most of the input electric power will be dissipated by the coil resistance. So an appropriate intermediate value must be selected for $\text{Re}(ZAA)$ for the sake of a high efficiency operation of the compressor. Although Refs. [2,3] has determined the correct value of $\text{Re}(ZAA)$ for a maximum-efficiency operation of

the compressor, the authors here re-examine the same issue using the energy considerations, thereby providing a clear physical meaning.

Before continuing with the next step, we define a characteristic constant of the compressor

$$Y = (BL)^2 / R_m / R_e \quad (12)$$

Defining the ratio between $\text{Re}(ZAA)$ and R_m as dimensionless acoustic resistance y , Eq. (10) becomes

$$\frac{1}{2} \text{Re}(\hat{E} \hat{I}^*) = \frac{1}{2} |\hat{I}|^2 \frac{Y}{y+1} R_e + \frac{1}{2} |\hat{I}|^2 R_e \quad (13)$$

So the efficiency of converting the electric power to electro-magnetic work is

$$\eta_1 = \frac{Y}{Y+y+1} \quad (14)$$

Meanwhile, according to Eq. (6), the conversion efficiency of electro-magnetic power into acoustic power is

$$\eta_2 = \frac{y}{y+1} \quad (15)$$

Multiplying both efficiencies exactly gives the compressor efficiency of converting electric power into acoustic power to drive the cold-head

$$\eta = \eta_1 \eta_2 = \frac{Y}{Y+y+1} \frac{y}{y+1} = \frac{Y}{Y+2+y+(Y+1)/y} \quad (16)$$

For a given compressor's Y , the maximum efficiency can only be reached when

$$y_{\text{best}} = \sqrt{Y+1} \quad (17)$$

and maximum available efficiency of the compressor is

$$\eta_{\text{max}} = \frac{Y}{Y+2+2\sqrt{Y+1}} = \frac{\sqrt{Y+1}-1}{\sqrt{Y+1}+1} \quad (18)$$

which is the same as in Refs. [2,3]

It is very interesting to see how the Y value changes the compressor's intrinsic maximum efficiency and how the dimensionless acoustic resistance y changes the available compressor efficiency.

Fig. 2 gives the dependence of the maximum compressor efficiency on the Y value. It can be seen that the maximum compressor efficiency increases with Y . When Y is small, say, below 50, the curve is very steep. After that, the steepness quickly drops. Specially, when Y is greater than 100, doubling Y to 200 only leads to a slight increase of efficiency by about 5%. While this doubling of Y means tremendous effort in order to improve the compressor

design, the gain may only be important to some special applications, e.g., space applications.

It has been pointed out that whether the maximum efficiency could be reached depends on the value of dimensionless acoustic resistance y . For this purpose, we define a ratio

$$r = y / \sqrt{Y+1} \quad (19)$$

When r equals 1, the compressor operates at its best efficiency according to Eq. (17). Fig. 3 displays how the ratio r influences the compressor efficiency with different Y values. Roughly speaking, near the optimum region ($r \approx 0.7-1.5$), the efficiency changes mildly around peak values by only a few percent and, the higher Y , the smaller the change. This actually provides two benefits. One is that the value of R_m needs not to be very accurate in order to give a good design of the system. A rough estimate based on practical measurements is usually sufficient for a good initial design of the cryocooler system with reasonable efficiencies. The other benefit is related to maximizing the power capability of the compressor, as will be discussed in the next sub-section.

3.2. Impedance match for maximizing the power output of a given compressor

Besides the efficiency of the compressor, another issue to be considered for a given compressor is that how to maximize the power output of the compressor, i.e. the acoustic power. Linear compressors generally have a limited current amplitude due to coil safety requirements and a limited piston displacement mainly due to flexure bearing lifetime considerations. The current limit is a little bit ambiguous because it is related to the cooling condition for the motor housing. The electro-magnetic work, most of which will go to the cold-head in the form of acoustic power in the case of a good impedance match, will be maximized when the current and displacement reach their respective limits simultaneously. Then from Eq. (9) and by assuming a mechanical resonance condition, the acoustic resistance $\text{Re}(ZAA)$ must satisfy

$$\frac{\dot{I}_{\text{max}} BL}{\omega \hat{x}_{\text{max}}} = \text{Re}(ZAA) + R_m \quad (20)$$

Although the conversion efficiency of electro-magnetic work into acoustic power should be considered, this criteria roughly ensures that the acoustic power is nearly maximized with a reasonable value of $\text{Re}(ZAA)$, which is typically 5–10 times larger than R_m . This actually sets the power criteria for coupling an existing linear compressor with the cold-head. If not matched, either the current or the piston displacement reaches their respective limit first, and thus the compressor power capability may not be fully utilized. Depending on the compressor parameters, the value of $\text{Re}(ZAA)$ from Eq. (20) may contradict the requirement of maxi-

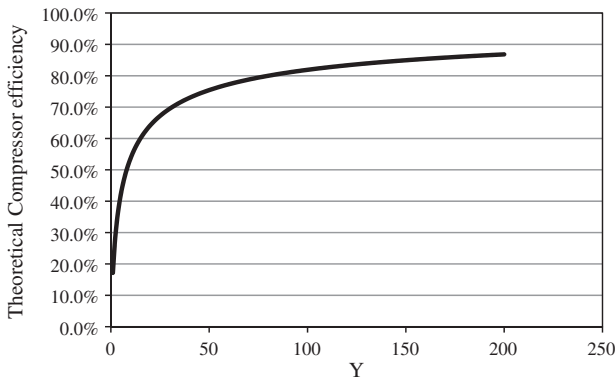


Fig. 2. Dependence of compressor maximum efficiency on Y .

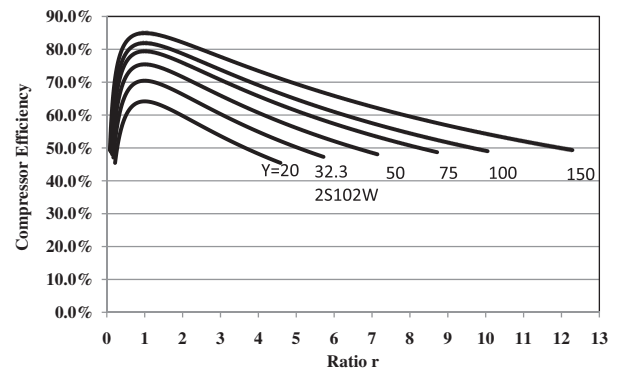


Fig. 3. Dependence of compressor efficiency on $y/\sqrt{Y+1}$.

imum efficiency, i.e., Eq. (17). In some cases, the contradiction may be serious, e.g., when we want a compressor designed for 60 Hz operation to operate at a 100 Hz. However, if the value of $\text{Re}(ZAA)$ according to Eq. (20) does not deviate much from the value according to Eq. (17), the phenomenon in Fig. 3 which we have discussed in the last paragraph of Section 3.1 actually means that the acoustic load could be adjusted with some flexibility without sacrificing the efficiency substantially but the available power may be significantly influenced.

On the other hand, if the mission is to design a compressor to match an existing cold-head, there is sufficient flexibility to set the current limit and displacement limit so that the power criteria Eq. (20) and the efficiency criteria Eq. (17) could be satisfied simultaneously.

3.3. The issue with adjusting piston diameter for impedance match

Generally speaking, the piston diameter plays a key role for the impedance match discussed above. It strongly influences both the real and imaginary part of ZAA, and if we try to adjust the piston diameter for high efficiency or maximum power output or both, the equivalent gas spring at piston front side is also changed, which will significantly impact on the mechanical resonance condition. In this case, one may require other means such as adjusting the moving mass or mechanical springs to return the compressor back to a resonance condition at the designed frequency.

4. Practical case study

In this section, we describe the use of a CFIC 2S102W compressor to study the two different cases: one is to find a matched pulse tube cold-head for 60 Hz operation, and the other is related to coupling a pulse tube cold-head designed for operation at around 100 Hz with this compressor. The 2S102W parameters, provided by the manufacturer and designed for 60 Hz operation [6], are listed in Table 1. The mechanical resistance R_m is measured under the pressurized conditions. It should be mentioned here, that the model 2S102W is of a dual-opposed piston configuration. The following will first investigate how the impedance Z seen by a single piston affects the efficiency and power of a single side. For the whole compressor, efficiency is the same and the power is simply doubled. Meanwhile, cold-head impedance Z_{ptc} is related to Z through

$$\frac{2}{Z} = \frac{1}{Z_{ptc}} + \frac{i\omega V_{void}}{\gamma P_0} \quad (21)$$

where V_{void} is the dead volume between the two pistons and inlet of the cold-head, P_0 is average pressure and γ is specific heat ratio.

4.1. Case study A: 2S102W at 60 Hz

Fig. 3 has given the corresponding curve for the 2S102W compressor operating at 60 Hz. The theoretical maximum efficiency for this compressor is 70.5%. To reach this efficiency, the acoustic resistance should be

$$\text{Re}(ZAA) = \sqrt{Y + 1} R_m = \sqrt{32.3 + 1} * 12 = 69.2 \quad (22)$$

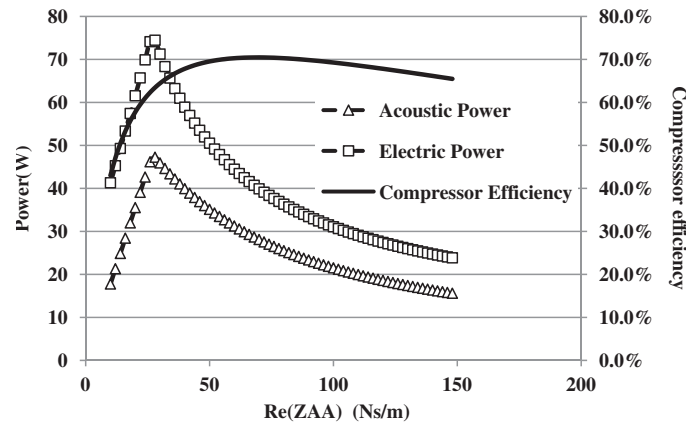


Fig. 4. Dependence of electric power, available acoustic power and compressor efficiency on $\text{Re}(ZAA)$ at 60 Hz.

From the power criteria defined in Eq. (18),

$$\text{Re}(ZAA) = \frac{\hat{I}_{\max} BL}{\omega \hat{X}_{\max}} - R_m = \frac{1.4 * 51.97}{2.0 * \pi * 60 * 0.005} - 12 = 27 \quad (23)$$

If $\text{Re}(ZAA)$ is smaller than 27 Ns/m, the displacement limit is reached first. Otherwise the current reaches its limit first. Fig. 4 gives a clearer picture of how the input electric power, available acoustic power and the compressor efficiency changes with $\text{Re}(ZAA)$. A maximum acoustic power of 47.7 W into the cold-head is achieved when $\text{Re}(ZAA)$ is equal to 27 Ns/m and the compressor efficiency is 63.1%, a value slightly lower than the maximum efficiency of 70.5% that is achieved when $\text{Re}(ZAA)$ is equal to 69.2 Ns/m. However, at this point the available acoustic power is larger by roughly 68% compared to the value of 28.3 W when $\text{Re}(ZAA)$ is equal to 69.2 Ns/m.

Meanwhile from Eq. (8), we can see that

$$\begin{aligned} \text{Im}(ZAA) &= 36,900 / (2 * \pi * 60) - 0.448 * 2 * \pi * 60 \\ &= -70.9 \end{aligned} \quad (24)$$

If we choose a value of 27 Ns/m for the real part and -70.9 Ns/m for the imaginary part of ZAA, the load impedance seen by a single piston is $6.0e7 - 15.8e7 * i$ Ns/m⁵. With an average pressure of 2.0 MPa, 60 Hz operation and a void volume of 30 cc in between, the cold-head acoustic impedance, according to Eq. (21), is $5.5e7 - 10.0e7 * i$ Ns/m⁵. This impedance value means that the pressure wave lags by 61.2° behind the volume flow rate at the inlet of the pulse tube cooler, which approximates the ideal value for the pulse tube cryocooler in the range of, say, 40–60°. If the void volume is increased further, the acoustic impedance could be improved with a smaller phase lag (e.g. 45° phase lag corresponding to 60 cc void volume), i.e., a better match with a normal pulse tube cryocooler could be realized. On the other hand, if the compressor could be re-designed, the ratio between the current limit (or more accurately, the product of BL and the current limit) and displacement limit may be increased to some extent to simultaneously provide a better efficiency and power capability.

Table 1
Parameters for a single motor of 2S102W.

BL (N/A)	$\text{Re}(\Omega)$	L_e (mH)	M (kg)	K (kN/m)	R_m (Ns/m)	D (mm)	$I_{\max}$ (A)	$X_{\max}$ (mm)	Void volume at front side (cc)
51.97	6.97	83.2	0.448	36.9 ^a	12 ^b	29.2	1.4	5	30

^a Spring constant as provided by CFIC is 35.1 kN/m, gas spring backside of the piston is 1.8 kN/m with 2.0 MPa helium gas.

^b Measurement with high pressure helium gas inside, an approximate number.

One more interesting feature is that once we select the acoustic impedance value, the pressure wave amplitude available to the pulse tube cryocooler is also fixed. For example, with the Z value of $6.0\text{e}7\text{--}15.8\text{e}7\text{ i Ns/m}^5$ according to power criteria, we have

$$\dot{p} = |\omega x_{\max} A Z| = 2.12 \text{ bar} \quad (25)$$

This value is only weakly related to the average pressure through the influence on the backside gas spring and mechanical resistance R_m .

4.2. Case study B: 2S102W at 100 Hz

Previously, we have built an in-line type 100 Hz cooler for operation at around 80 K. At that time, we did not have a suitable compressor but rather used the 2S102W compressor that was available in our lab. Besides the question of whether the compressor could be resonated at around 100 Hz, we also wanted to know the compressor efficiency and the available acoustic power at this frequency. Although the electro-magnetic power is proportional to the frequency if the current and displacement limits could be reached simultaneously, the available acoustic power could be a different story after the conversion efficiency is considered.

With the same parameters and R_m values, $\text{Re}(ZAA)$ should still be 69.2 Ns/m for a maximum compressor efficiency of 70.5%. However, from the power criteria defined by Eq. (20), the required value of $\text{Re}(ZAA)$ is only 11.4 Ns/m due to a much increased frequency, which corresponds to a very low compressor efficiency of 45.9%. For this reason, the value of $\text{Re}(ZAA)$ should be much higher than 11.4 Ns/m for a reasonable compressor efficiency, and this definitely means that current limit is always reached far before the displacement limit is reached.

Fig. 5 displays the dependence of the input electric power, available acoustic power and the compressor efficiency on the value of $\text{Re}(ZAA)$ when the working frequency is 100 Hz. The acoustic power reaches a maximum of 56.2 W when $\text{Re}(ZAA)$ is equal to 11.4 Ns/m, slightly larger than the maximum value of 47.7 W at 60 Hz. But the input electric power is much larger due to a significantly decreased compressor efficiency. If a better compressor efficiency is needed, the available acoustic power will decrease. Obviously, the output acoustic power from a compressor designed for a certain frequency cannot be simply proportionally increased by the frequency. The impedance match requirement concerning efficiency, current limit and displacement limit should be carefully evaluated.

Using impedance values for a real cooler in our lab, we take one more look at the practical coupling goes. Roughly speaking,

the cryocooler has a 10 mm i.d. regenerator and 6 mm i.d. pulse tube, which gives a typical impedance value of $9.4\text{e}8\text{--}5.6\text{e}8\text{ i Ns/m}^5$ at an average pressure of 3.5 MPa and 100 Hz. The impedance Z seen by a single piston according to Eq. (20) is therefore $1.1\text{e}8\text{--}5.2\text{e}8\text{ i Ns/m}^5$, which leads to a resonance frequency of around 102 Hz according to Eq. (8). $\text{Re}(ZAA)$ is 49.4 Ns/m which means that a compressor efficiency of 69.6% could be achieved while the displacement amplitude only reaches 1.8 mm when the current reaches the limit of 1.0 Arms. The available acoustic power is 34.9 W for a single piston. The pressure wave amplitude for the cold-head is 4.18 bar when the current reaches 1 Arms limit under this condition.

One should remember that the above discussion holds when the moving mass is at mechanical resonance. Off-resonance will lead to a further decreased efficiency and the current limit will be reached quicker.

As a matter of fact, with an average pressure of 3.5 MPa, 100 Hz frequency and a pressure ratio over 1.25, the R_m value could be much larger than 12 Ns/m, as evidenced from our experiences. Such an increase produces a more serious influence on the impedance match and the associated compressor efficiency and available acoustic power. Since this paper is not intended to cover all the cases, but to present a methodology for impedance match, we will not discuss this issue further.

5. Conclusion

In this paper, the impedance match between the cold-head and the linear compressor in Stirling type cryocoolers has been carefully analyzed. Guiding principles concerning the efficiency, the output acoustic power and mechanical resonance have been generalized. For a given compressor, sometimes a compromise should be made between the efficiency and output power capability when selecting the acoustic impedance. Furthermore, if the compressor's working frequency deviates significantly from its original design, additional care should be taken.

As addressed at the beginning of the paper, non-linear effects may lead to the modifications of the models used here. The practical case studies are also meant to provide a deeper understanding about the impedance match. From our experiences in the past few years, the methodology used here is rather useful for a good initial design of the cryocooler system. Subsequently, some higher-order numeric models, as well as the experimental tuning, will be necessary for a refined design.

Acknowledgement

This work is financially supported by the National Natural Science Foundation of China under Contract Number of 50890181 and the National Basic Research Program of China (No. 2010CB227303).

References

- [1] Swift GW. Thermoacoustics: a unifying perspective for some engines and refrigerator. Sewickley (PA): Acoustic Society of America Publications; 2002.
- [2] Wakeland Ray Scott. Use of electrodynamic drivers in thermoacoustic refrigerators. J Acoust Soc Am 2000;107(2):827–32.
- [3] Zhanghua Wu. Theoretical and experimental study of electrically driven traveling-wave thermoacoustic refrigerator in room temperature range. Ph. D thesis, Chinese Academy of Sciences; 2006.
- [4] Reed J, Bailey PB, Daddl MW, Davis T. Motor and thermodynamic losses in linear cryocooler compressors. Adv Cryog Eng 2006;51:361–8.
- [5] Robert Redlich, Reuven Unger, Nicholas van der Walt. Linear compressors: motor configuration, modulation and systems. In: International compressor engineering conference, Purdue University, July 23–26, 1996.
- [6] Operation manual for 2S102W pressure wave generator. CFIC Inc.; 2006.

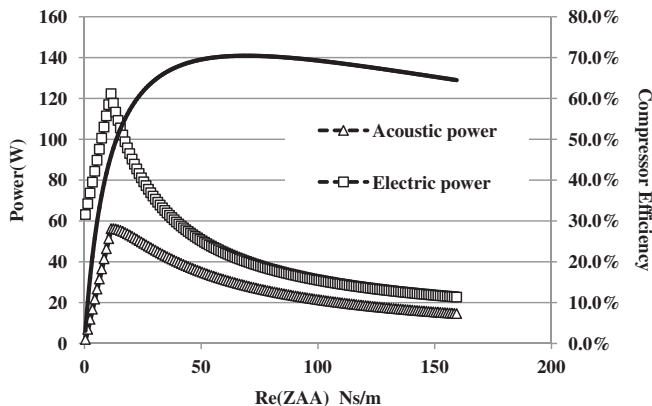


Fig. 5. Dependence of electric power, available acoustic power and compressor efficiency on $\text{Re}(ZAA)$ at 100 Hz.